Minitest N°4 ALT ⟶ Corrigé

Soient f et g définie sur $\mathbb{R}^2$ par $f(x,y) = x^2e^{xy}$ et $g(x,y,z) = (5x^2 - 3xz \ ; \ (z+y)^2 \ ; \ x - y + z)$

Calculer :

a) [1pt] $\nabla(f)$.

b) [1pt] $\Delta(f)$.

c) [1pt] $\text{div}(g)$.

d) [1pt] $\text{rot}(g)$.

e) [1pt] $H_f(x,y)$.

Corrigé

a) $\nabla(f) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial x} \end{bmatrix} = \begin{bmatrix} 2xe^{xy} + x^2ye^{xy} \\ x^2ye^{xy} \end{bmatrix} = \begin{bmatrix} (2x + x^2y)e^{xy} \\ x^2ye^{xy} \end{bmatrix}$

b) $\Delta(f) = \dfrac{\partial f^2}{\partial x^2} + \dfrac{\partial f^2}{\partial y} = (2 + 2xy)e^{xy} + y(2x + x^2y)e^{xy} \quad + \quad x^4e^{xy} \quad = (2 + 4xy + x^2y^2 + x^4)e^{xy}$

c) $\text{div}(g) = \dfrac{\partial g_1}{\partial x} + \dfrac{\partial g_2}{\partial y} + \dfrac{\partial g_3}{\partial z} = 10x - 3z \quad + \quad 2(z+y) \quad + \quad 1 \quad = 10x - z + 2y + 1$

d) $\text{rot}(g) = \begin{bmatrix} \frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z} \\ \frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x} \\ \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y} \end{bmatrix} = \begin{bmatrix} -1 - 2(y+z) \\ -3x - 1 \\ 0 - 0 \end{bmatrix}$

e) $H_f(x,y) = \begin{bmatrix} \dfrac{\partial f^2}{\partial x^2} & \dfrac{\partial f^2}{\partial y \partial y} \\ \dfrac{\partial f^2}{\partial x \partial y} & \dfrac{\partial f^2}{\partial y^2} \end{bmatrix} = \begin{bmatrix} (2 + 4xy + x^2y^2)e^{xy} & (3x^2 + x^3y)e^{xy} \\ (3x^2 + x^3y)e^{xy} & x^4e^{xy} \end{bmatrix}$